

**NASA TECHNICAL  
MEMORANDUM**

**N 7 1 - 2 7 8 6 7**

**NASA TM X-67860**

**NASA TM X-67860**

**CASE FILE  
COPY**

**INTERFACIAL SHEAR FACTORS FOR LAMINAR  
FILM BOILING FROM A VERTICAL SURFACE**

by Kenneth J. Baumeister and Harold R. Jacobs  
Lewis Research Center  
Cleveland, Ohio

**TECHNICAL PAPER** proposed for presentation at  
Thirteenth International Congress of Refrigeration  
Washington, D. C., August 27-September 3, 1971

# INTERFACIAL SHEAR FACTORS FOR LAMINAR FILM

## BOILING FROM A VERTICAL SURFACE

by

Kenneth J. Baumeister  
NASA Lewis Research Center  
Cleveland, Ohio

and

Harold R. Jacobs  
University of Utah  
Salt Lake City, Utah

### SUMMARY

Forced flow film boiling heat transfer coefficients from a vertical surface have been expressed in terms of the interfacial shear or the so called "slip" at the liquid vapor interface. The heat transfer coefficient is expressed as a function of the Froude number and the shear factor. The shear factors are determined from an exact analysis which accounts for the coupling between the vapor and liquid momentum at the interface. The correlating equation may be useful to the designer interested in a simple correlating technique valid over a wide range of Froude Numbers.

### INTRODUCTION

Film boiling heat transfer plays an important role in energy transfer in cryogenic systems. In this paper film boiling from a vertical surface will be studied. In particular, we will study how the Froude number and vapor and liquid properties effect the interfacial shear or "slip" at the boiling interface. A knowledge of the interfacial shear is important for high pressure cryogenics especially in the vicinity of the thermodynamic critical point.

The earliest heat transfer analysis of pool film boiling from a vertical surface was by Bromley (ref. 1). Later, Hsu and Westwater (ref. 2) extended Bromley's analysis for both laminar and turbulent vapor flow. Bromley con-

sidered the equations of motion in the hot vapor film with the boundary conditions at the wall equal to

$$y = 0 \quad u = 0 \quad (1)$$

Unfortunately, the boundary condition at the liquid vapor interface,  $\delta$ , shown in Figure 1 case (a), was an unknown. Bromley, however, considered two limiting possibilities on the velocity at the liquid vapor interface. The first was that (fig. 1(b))

$$y = \delta_S \quad u = 0 \quad (2)$$

that is, the vapor does not drag any liquid along. The second possibility considered was (fig. 1(c))

$$y = \delta_M \quad \frac{\partial u}{\partial y} = 0 \quad (3)$$

that is, the shear between the liquid and vapor is zero. Bromley derived a simple expression for the heat transfer coefficient for each of these two assessed boundary conditions. These expressions bracketed the unknown heat transfer coefficient. Bromley then suggested that a heat transfer coefficient which is the average of these two extremes be used as a correlating equation.

Koh (refs. 3 and 4) extended Bromley's analysis by considering the momentum transfer between the liquid and the vapor. Koh's results are expressed numerically in terms of the important parameters such as Prandtl number and the density viscosity ratios. From his analysis, Koh showed that for

$$\left( \frac{\rho_V \mu_V}{\rho_L \mu_L} \right)^{1/2} < 0.01 \quad (4)$$

boundary condition (eq. (2)) is a good choice. Equation (4) generally holds for low pressure boiling. A notable exception to equation (4) is film boiling near the thermodynamic critical point.

Later forced flow past a vertical surface in film boiling was studied

(figs. 1(d) and (e)). Rankin considered the equations of motion in the vapor and at the liquid vapor interface, he assumed

$$y = \delta \quad u = u_{\infty} \quad (5)$$

that is, the interface moves at the free stream velocity. For high liquid velocities this is a reasonable assumption if  $\mu_L \gg \mu_V$  (see ref. 6). For a low liquid velocity or near the critical point, however, this is only an approximation as seen from the flow profile shown in Figure 1(d).

Recently Jacobs and Boehm (ref. 6) extended the theory for the case of film boiling from vertical surfaces with flow. They coupled the governing equations of motion and energy in both the vapor film and surrounding liquid. In this case, the numerical solutions of the heat transfer coefficients are correlated as a function of Froude number as well as other system parameters included in Koh's (refs. 3 and 4) pool boiling analysis. The numerical results in reference 6, however, are limited to a single curve at one density-viscosity ratio.

The purpose of the present paper is to present a relatively simple heat transfer correlation for flow film boiling past a vertical surface. First, a shear parameter  $K$  is defined; then, the equations of motion and energy in the hot vapor are solved with the factor  $K$  treated as an unknown parameter. Finally, by a comparison with the exact numerical results in reference 6 the actual  $K$  is specified. The simple heat transfer correlation which results may be of practical use to the design engineer.

#### LIST OF SYMBOLS

$C_p$	specific heat at constant vapor pressure
$Fr$	Froude Number $u_{\infty}^2/gx$
$Gr$	Grashof Number $\rho_V(\rho_L - \rho_V)gx^3/\mu_V^2$
$g$	coefficient of gravity

$g_c$	gravitational constant
$h$	heat transfer coefficient
$h^*$	dimensionless $h$ defined by equation (12)
$K$	shear factor, equation (10)
$k_v$	thermal conductivity of vapor
$Nu$	Nusselt number $hx/k_v$
$P$	pressure
$Pr$	Prandtl number $\mu C_p/k_v$
$S$	defined by $C_p \Delta T / \lambda Pr$
$T_w$	wall temperature
$T_s$	saturation temperature
$\Delta T$	$(T_w - T_s)$
$u$	velocity parallel to surface
$u_\infty$	free stream liquid velocity
$u_{max, pool}$	see equation (11)
$W$	mass flow rate per unit length
$x$	distance along plate from leading edge
$y$	distance normal to surface
$\delta$	vapor gap thickness
$\delta_S$	vapor gap thickness for complete slip ( $K = 0$ )
$\delta_M$	vapor gap thickness for no slip ( $K = 1$ )
$\lambda$	latent heat of vaporization
$\mu_L$	viscosity of liquid
$\mu_v$	viscosity of vapor
$\sigma$	surface tension

### Theory

The following analysis for laminar film boiling along a vertical surface

assumes a smooth laminar film, constant pressure gradient, constant vapor properties which are evaluated at the film temperature, and negligible fluid inertia. In the energy analysis, heat is assumed to be transferred only by molecular conduction. The liquid is assumed to be at its saturation temperature. Consequently, the governing momentum and energy equations for the vapor film shown in Figure 1 are as follows:

$$\frac{\partial^2 u}{\partial y^2} - \frac{\epsilon_c}{\mu v} \frac{\partial P}{\partial x} = 0 \quad (5)$$

$$q = \frac{k_v \Delta T}{\delta} = h \Delta T \quad (6)$$

$$W = \int_0^{\delta} \rho_v u \, dy \quad (7)$$

Equation (5) is the momentum equation; equation (6) is the energy equation; and equation (7) accounts for the increased mass vapor flow due to vaporization.

The boundary conditions on the energy equation are simply

$$y = 0 \quad T = T_w; \quad y = \delta \quad T = T_{sat} \quad (8)$$

The boundary condition on the momentum equation at the wall is given by equation (1). The boundary condition at the liquid vapor interface, however, needs special consideration, and it is here that we introduce the slip ratio.

#### Shear Factor K

For pool boiling, the no shear boundary condition, equation (3), leads to a predicted vapor velocity which is the maximum possible obtainable at the liquid vapor interface,  $u_{max,pool}$ . However, in flow boiling at high Froude numbers the free stream velocity  $u_{\infty}$  controls the interfacial velocity. Consequently, because of interfacial shear, the actual interfacial vapor velocity is within the range

$$0 < u(\delta) < u_{\infty} + u_{max,pool} \quad (9)$$

For high Fr numbers,  $u_\infty$  dominates while at low Fr numbers or pool boiling  $u_{\max, \text{pool}}$  dominates. From these considerations, the choice of a boundary condition at the liquid vapor interface would be

$$y = \delta \quad u = K(u_\infty + u_{\max, \text{pool}}) \quad (10)$$

where K is a shear factor which depends on the magnitude of  $u_\infty$ , fluid properties, and position x. It can be readily shown (ref. 1) that

$$u_{\max, \text{pool}}^{(\delta)} = \frac{(\rho_L - \rho_V)g}{\mu_V} \left[ \frac{k_V \Delta T x \mu_V}{\lambda \rho_V (\rho_L - \rho_V) g} \right]^{1/2} \quad (11)$$

#### Solution for the Heat Transfer Coefficient

The solution of equations (5), (6), and (7) subject to boundary conditions (8), (1), and (10) yields the following expression for the film boiling heat transfer coefficient written in terms of Froude number and the factor K

$$h^* = \frac{Nu}{\left[ \frac{Gr}{S} \right]^{1/4}} = \frac{.5}{\left[ \left( \frac{9}{16} K^2 \left\{ 1 + \frac{2}{3} \left[ \frac{\rho_V Fr}{(\rho_L - \rho_V) S} \right]^{1/2} \right\}^2 + 1 \right)^{1/2} - \frac{3}{4} K \left\{ 1 + \frac{2}{3} \left[ \frac{\rho_V Fr}{(\rho_L - \rho_V) S} \right]^{1/2} \right\} \right]^{1/2}} \quad (12)$$

Because of paper length restrictions, the derivation is not given here. However, the integrations are simple and the integration steps follow in a similar manner to that given in reference 5 for the no slip case.

In the limiting case of pool boiling ( $u_\infty = 0$  or  $Fr = 0$ ), for maximum shear ( $K = 0$ )  $h^* = 0.5$ , while for no shear ( $K = 1$ )  $h^* = 0.707$ . These results agree with the earlier work of Bromley.

Solving equation (12) for the factor K yields

$$K = \frac{1 - \left(\frac{.25}{h^{*2}}\right)^2}{\frac{3}{2} \left(\frac{.25}{h^{*2}}\right) \left\{ 1 + \frac{2}{3} \left[ \frac{\rho_V Fr}{(\rho_L - \rho_V) S} \right]^{1/2} \right\}} \quad (13)$$

#### DISCUSSION

In reference 6 a single curve is presented for a dimensionless heat transfer coefficient as a function of Fr number for  $S = 6 \times 10^{-4}$ ,  $\rho_L/\rho_V = 100$  and  $\mu_L/\mu_V = 25$ . A plot of the factor K for this curve is shown in Figure 2 as function of Froude number. As seen in Figure 2, for Froude numbers less than 0.001 the K parameter approaches a constant value of 0.59, while for Froude number greater than 100, K reaches a value near 0.75.

Near a Fr number of 0.01 a minimum in K is seen. This dip could be due to the manner in which K is defined in the low velocity region, where  $u_{\infty}$  and  $u_{\max, \text{pool}}$  are of the same order.

The asymptotic value of K at large Froude numbers for this case was 0.75, because of the small value of S assumed in reference 6. In general, however, K will approach an asymptotic value of 1 for large Froude numbers whenever the inequality given by equation (1) hold true, as shown in Figure 2.

At the present time more exact numerical solutions are required to map K over a large range of density and viscosity ratios especially near the thermodynamic critical point where the density-viscosity ratio approaches one. Next, a proper set of dimensional groups should be derived such that all the curves could be reduced to a single curve.



## REFERENCES

1. Bromley, LeRoy A.: Heat Transfer in Stable Film Boiling. Chem. Eng. Prog., vol. 46, no. 5, May 1950, pp. 221-227.
2. Hsu, Y. Y.; and Westwater, J. W.: Approximate Theory for Film Boiling on Vertical Surfaces. Chem. Eng. Prog. Symp. Ser., vol. 56, no. 30, 1960, pp. 15-24.
3. Koh, J. C. Y.: Analysis of Film Boiling on Vertical Surfaces. J. Heat Transfer, vol. 84, no. 1, Feb. 1962, pp. 55-62.
4. Koh, James C. Y.: Simultaneous Convection and Radiation in Laminary Film Boiling on Vertical Surface. Paper 63-HT-2, ASME, Aug. 1963.
5. Rankin, Sidney: Force Convection Film Boiling Inside of Vertical Pipes. Ph.D. Thesis, University of Delaware, 1960.
6. Jacobs, H. R.; and Boehm, R. F.: An Analysis of the Effects of Body Force and Forced Convection on Film Boiling. Heat Transfer 1970. Vol. 5. Ulrich Grigull and Erich Hahne, eds., Elsevier Publ. Co., 1970, pp. B3.9.1-B3.9.11.

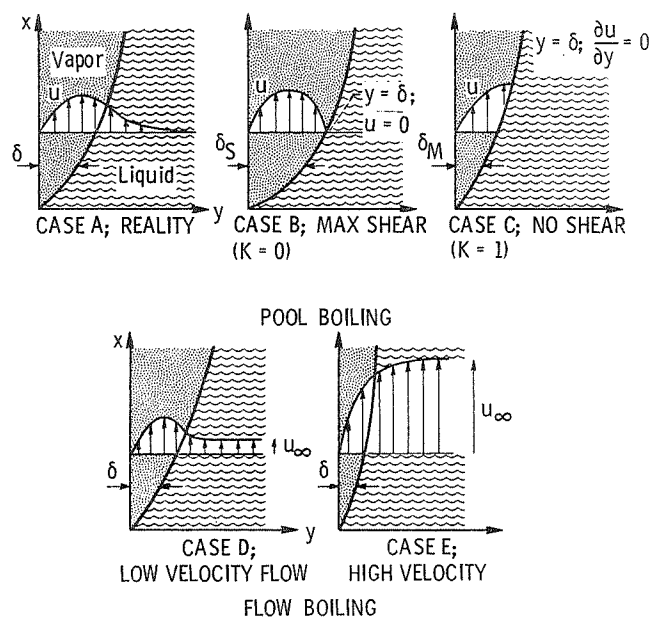
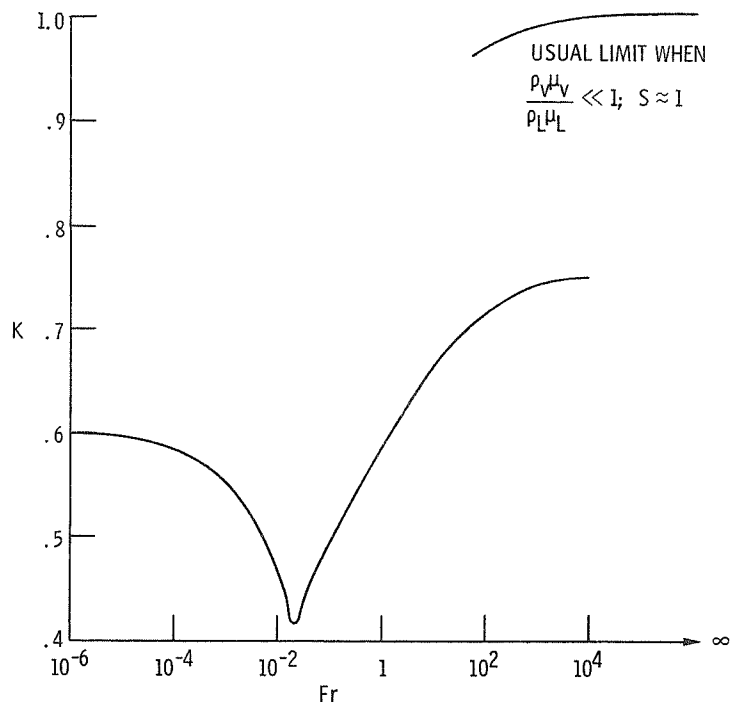


Figure 1. - Boundary conditions in pool film boiling.

Figure 2. - Slip factor  $K$  as a function of Froude number for  $S = 6 \times 10^{-4}$ ,  $\mu_L / \mu_V = 25$  and  $\rho_L / \rho_V = 100$ .